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A Noise-Smoothing Filter Based on Adaptive Windowing and Its Application to Penumbral Imaging


Abstract

Penumbral imaging is a technique which uses the facts that spatial information can be recovered from the shadow or penumbra that an unknown source casts through a simple large circular aperture. The technique is based on a linear deconvolution. In this paper, a two-step method is proposed for decoding penumbral images. First a local-statistic filter based on adaptive windowing is applied to smooth the noise; then, followed by the conventional linear deconvolution. The simulation results show that the reconstructed image is dramatically improved in comparison to that without the noise-smoothing filtering, and the proposed method is also applied to real experimental x-ray imaging.

Key words: penumbral imaging, local-statistic filter, adaptive windowing, signal-dependant noise

1. Introduction

Penumbral imaging[1], one of CAI (coded aperture imaging) techniques, is proposed for imaging objects that emit high-energy photons, where such objects arise, for example, in nuclear medicine, X-ray astronomy, and laser fusion studies. For these high-energy photons, classical imaging techniques (e.g., lens) are not applicable.

The penumbral aperture is extremely simple, being just a large circular aperture. The spatial information of an unknown source can be recovered from the shadow or penumbra casted by the source. Since such an aperture can be "drilled" through a substrate of almost any thickness, the technique can be easily applied to highly penetrating radiations such as neutrons and γ rays. To date, the penumbral imaging technique has been successfully applied to image the high-energy x-rays [1], protons [2] and neutrons [3], [4] in laser fusion experiments.

Since penumbral images are always degraded by noise, a Wiener filter [2], [3], [4], where the mean-square error is minimized, is used for decoding process. Though the Wiener filter is a powerful technique for noise minimization, it is impossible to avoid reduction of resolution and quality of the reconstructed image.

In this paper, we proposed a two-step technique for decoding penumbral images. We first use a local-statistic filter based on adaptive windowing to smooth the noise; then, follow it with the conventional Wiener filter for decoding. The basic concept of the penumbral imaging is given in Sec.2, the noise-smoothing filter based on adaptive windowing is discussed in Sec.3, the simulation results are presented in Sec.4, and the experimental results are given in Sec.5.

2. Penumbral Imaging

The basic concept of the penumbral coded
aperture imaging technique is shown in Fig. 1. The encoded image consists of a uniformly bright region surrounded by a penumbra. Information about the source is encoded in this penumbra. It is known that the encoded image $P(r)$ is given by [1], [5]

$$P(r) = A\left(-\frac{L_1}{L_1 + L_2} r\right) * O\left(-\frac{L_2}{L_2} r\right)$$

(1)

where $A(r)$ is the aperture function or point spread function (PSF); $O(r)$ is the function describing the source; $L_1$ and $L_2$ are the distances from source to aperture and from aperture to detector, respectively; $L_2/L_1$ is the magnification of the camera; and $*$ denotes the convolution. Thus given $P(r)$, $A(r)$, and $L_2/L_1$, the source function $O(r)$ may be deconvolved. In general, deconvolution techniques are very sensitive to the noise contained in the encoded image, because the noise will be amplified to very high levels at higher spatial frequency domain, where the amplitude closes into zero. Here a Wiener filter, where the mean-square error is minimized, is usually used for deconvolution. The Wiener filter $W(u)$ defined in the Fourier transform domain is given as [5]

$$W(u) = \frac{1}{A_F(u)} \cdot \left(1 + \frac{\Gamma}{|A_F(u)|^2}\right)^{-1}$$

(2)

where $u$ is the spatial frequency variables and $A_F(u)$ is the Fourier transform of PSF. $\Gamma$ is a constant proportional to the noise-to-signal power density ratio. If $\Gamma=0$, the filter is the inverse filter, and larger $\Gamma$ will significantly reduce the resolution at the higher spatial frequency domain.

If the image is with a stationary mean and variance, the Wiener filter is a powerful technique; while for penumbral imaging, since the information is only contained in the penumbra (the edge of the image), it is impossible to avoid reduction of resolution and quality of the reconstructed image. A new technique based on nonstationary mean and variance image model should be developed for reconstruction of penumbral images.

We proposed here a two-step technique for reconstruction of penumbral images. We first use a local-statistic filter based on adaptive windowing to smooth the noise; then, follow it with the conventional Wiener filter for reconstruction.

3. Noise-Smoothing Filter Based on Adaptive Windowing

The simplest method of smoothing is taking the sample mean or median from a running window. Although the running mean filter provides excellent noise suppression over slowly varying signals, it smears edges. Lee proposed a local
statistics algorithm to overcome this problem [6]; where the algorithm used both the sample mean and variance in a fixed window to give priority weighting to the pixel values to be estimated and hence suppress the effect of an edge at the window boundary.

Let the noisy image $y_{ij}$ at $(i,j)$ point be represented by the sum of the original image $x_{ij}$ and the noise $n_{ij}$ such that

$$y_{ij} = x_{ij} + n_{ij},$$

where $n_{ij}$ is uncorrelated additive white noise of zero-mean and variance $\sigma^2$. The estimate value by Lee's algorithm is given by

$$\hat{y}_{ij} = Z_{ij} + Q_{ij}(y_{ij} - Z_{ij}),$$

and

$$Q_{ij} = 1 - \frac{\sigma^2}{V_{ij}},$$

where $Z_{ij}$ and $V_{ij}$ are the sample local mean and variance in a fixed window, respectively. From Eqs.(4) and (5) we can see that for a flat or slowly varying region, $V_{ij} = \sigma^2$, resulting in $Q_{ij} = 0$ and $y_{ij} = Z_{ij}$. This means that the estimated value is equal to local mean value. On the other hand, for an edge region, $V_{ij} >> \sigma^2$, resulting in $Q_{ij} = 1$ and $y_{ij} = y_{ij}$. This means that the estimated value is equal to the measured pixel value itself and the edge is preserved.

The limitation of Lee's algorithm is that for an edge region with lower SN ratio, the difference between $V_{ij}$ and $\sigma^2$ becomes smaller, resulting in $Q_{ij} \to 0$ and $y_{ij} \to Z_{ij}$. This means the edge will be smeared.

In order to overcome this limitation, we combined Lee's algorithm with an adaptive windowing method. The adaptive windowing method is based on that proposed by Song [7], where the window is expanded or contracted according to the computed value of the signal activity index. The determination of the adaptive window size is as follows.

First we roughly determine a maximum window size $L_{max} (=2N_{max} + 1)$. The sample statistics needed for generating the signal variance are computed over the maximum window centered at the filtering point $(i,j)$. We adopt square windows, since the square windows make the algorithm simple and isotropic [7]. Among the elements inside an $L_{max} \times L_{max}$ window, only the samples $F(i,j)$ on the boundary of window are used in measuring signal characteristics of pixel $(i,j)$.

Then it can be shown that the number of elements in $F(i,j)$ on the boundary of window is $4(L_{max} - 1)$ and that the sample mean $Z_{ij}$ and the mean of the squared data $R_{ij}$ for signal variance are given by

$$Z_{ij} = \frac{1}{4(L_{max} - 1)} \sum_{(k,l) \in F(i,j)} y_{kl},$$

$$R_{ij} = \frac{1}{4(L_{max} - 1)} \sum_{(k,l) \in F(i,j)} y_{kl}^2.$$

Then the sample variance for the signal variance becomes

$$V_{ij} = R_{ij} - Z_{ij}^2.$$

Since the sample variance consists of the signal variance and the noise variance, the signal
variance is defined as
\[ S_{ij} = \max \left( V_{ij} - \sigma^2, 0 \right) \]  \hspace{1cm} (9)

According to the computed signal variance Eq.(7), the window size \( N_{ij} \) is determined as:
\[ N_{ij} = \frac{\sigma^2}{\alpha \cdot S_{ij} + \sigma^2} \]  \hspace{1cm} (10)
\[ L_{ij} = 2N_{ij} + 1 \]  \hspace{1cm} (11)

where \( \alpha \) is a control parameter; and it is evident that the larger is the \( S_{ij} \), the smaller is the window size \( L_{ij} \).

4. Simulation Results

The performance of the proposed nois-smoothing filter based on adaptive windowing is evaluated by computer simulations. Figure 2(a) shows a typical penumbral image of "E" without noise. Figure 2(b) is the penumbral image with an additive Gaussian noise (\( \mu=0, \sigma=I_{\text{max}}/10 \)). Figures 2(c) and 2(d) show the image by the proposed filter and computed window size, respectively. It can be seen that the noise is sufficiently smoothed over, while the edge is preserved. The window size is significantly varied due to the signal variance.

In order to make a quantitative comparison, the mean squared error (MSE) is used as a measure, which is defined as:
\[ \epsilon = \frac{1}{N} \sum_{i \neq j} \left[ \hat{y}_{ij} - y_{ij} \right]^2 \]  \hspace{1cm} (12)

where, \( N \) is pixel number of the image. The comparison of the proposed filter with other filters is shown in Table 1. The window sizes

![Fig. 2 Penumbral images and their profiles. (a) without noise, (b) with a Gaussian noise, (c) after the proposed noise-smoothing filtering, and (d) computed window size for filtering.](image)

<table>
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<tr>
<th>Filter (size)</th>
<th>Mean (fixed: L=7)</th>
<th>Median (fixed: L=3)</th>
<th>Lee's method (fixed: L=7)</th>
<th>Proposed method (adaptive: L_{\text{max}}=15)</th>
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<td>( \epsilon )</td>
<td>2267.3</td>
<td>279.5</td>
<td>234.5</td>
<td>126.9</td>
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used were those which produced the least MSE. It is evident that the proposed filter performs the best.

The reconstructed images of Fig. 2 by the Wiener filter are shown in Fig. 3; Fig. 3(a) is the result of Fig. 2(a) (without noise); Figs. 3(b) and 3(c) are the results of Fig. 2(b) (with noise, but without filtering); and Fig. 3(d) is the result of Fig. 2(c) (with noise and with filtering). It can be seen that without the noise-smoothing filtering before the decoding, we have to use a larger $\Gamma$, resulting in blurring in the reconstructed image; while with the noise-smoothing filtering before the decoding, we can use a small $\Gamma$ because the SNR of the penumbral image is improved, and, consequently, the high quality reconstruction is obtained.

5. Experimental Results

We applied the proposed technique to x-ray imaging of laser-imploded targets with a penumbral camera. The experiments were carried out at the frequency doubled (0.53 $\mu$m) 12 beams Nd: glass laser facility, GEKKO XII [8] at Osaka University. The penumbral camera used a 530$\mu$m diameter hole drilled in a 25$\mu$m thick tantalum substrate. The irregularity of the hole from perfect circle was less than 1%. The distance between the aperture and the target was 11.6cm and the magnification of the camera was x8.8. An x-ray CCD camera (Princeton Instrument) [9] was used as a detector to record the encoded x-ray image. A titanium foil filter with a thickness of 100$\mu$m and a beryllium foil filter with a thickness of 40$\mu$m were placed in front of the CCD to obtain the encoded image with photon energies of 5keV and 10keV to 30keV taking account of the spectral response of the CCD camera [9].

A CH plastic shell target with a typical thickness of 8$\mu$m and a typical diameter of 530$\mu$m was irradiated by partially coherent laser lights (PCL) through random phase plates at a wavelength of 0.53-\mu m. The pulse shape of the laser lights consisted of a 1.6-ns squared pulse as a main pulse and a 200-ps prepulse with a time separation of 400-ps and the total laser energy was about 3kJ.

A typical penumbral image recorded by the CCD camera is shown in Fig. 4(a). It was divided into 100x100 pixels. The size of one pixel is about 111$\mu$m corresponding to a pixel...
resolution on the source plane of 12.6μm.

As shown in Fig.4(a), the encoded image is degraded with Poisson noise which is a signal-dependent noise [10]. Since the proposed noise smoothing filter does not consider the signal dependency, it is desirable to transform Poisson noise to signal-independent noise before applying the smoothing-filter. The Poisson distribution can be approximately expressed as a normal distribution using a transformation proposed by Anscombe [10], [11] as

$$P = 2(P + 3/8)^{1/2}$$  \hspace{1cm} (13)

We first employ the Anscombe transformation to make Poisson noise approximately signal-independent. Then the smoothing-filter is applied to the transformed image. The local variance is estimated as 960 from a running window of 40x40 pixels in the central flat region. After smoothing, the inverse Anscombe transformation is applied to the smoothed estimate. Figure 4(b) shows the image after the smoothing and the inverse Anscombe transformation.

Figure 5 shows the process of reconstruction. The reconstructed images without (Fig.4(a)) or with (Fig.4(b)) the proposed noise-smoothing filter (smoothing process) are shown in Figs.6(a) and 6(b), respectively. The $Γ$'s used in the Wiener filter for decoding are the same ($Γ = 0.15$) for both cases. The improvement of the reconstruction in the artifact level by the noise-smoothing filter is obvious. The size of the compressed core was estimated to be 22μm.

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**Experimental data**

(Penumbral image)

- Anscombe Trans.
- Estimation of $σ^2$
- Noise-smoothing Filter
- Inverse Anscombe Trans.
- Wiener Filter
- Reconstructed Image

**Fig. 5 Process of the reconstruction.**
which was almost the same result as that obtained from a pinhole image [9]. Discussion on the structure of compressed core is outside the scope of this paper and it will be done elsewhere.

6. Conclusion

In this paper, we proposed a noise-smoothing filter based on adaptive windowing for reconstruction of penumbral images. The simulation results and experimental results both show that the reconstructed image is improved in comparison to that without the noise-smoothing filtering.

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References


