The k—th power free kernels of sequence a_n^n+1

Akira TAKAKU*

1. Let k be an integer$\geqq 2$ and fix it. Let N be any positive integer and let $N=DX^k$. Then D is uniquely determined and called *the k—th powerfree kernel* of N. Let $\{a_n\}$ be any sequence of positive integers and $a_n\to\infty$ as $n\to\infty$. Then we prove following

THEOREM 1. *If $k\geqq 5$, then the sequence $\{a_n^n+1\}$ has infinitely many distinct k—th power free kernels.*

THEOREM 2. *Let $\{a_n\}$ be any sequence of even integers. Then the sequence $\{a_n^n+1\}$ has infinitely many distinct k—th power free kernels.*

In particular, for $k=2$ Theorem 2 shows that the odd sequence a_n^n+1 generates infinitely many real quadratic fields $\mathbf{Q}(\sqrt{a_n^n+1})$.

2. PROOF.

Proof of Theorem 1: Assume that there are a finite number of k—th power free kernels of a_n^n+1 when n runs all natural numbers. We denote them $D_1, D_2,\dots,D_s$. Let $p_1,\dots,p_t$ be all distinct prime divisors of $D_1,\dots,D_s$ and m any positive integer. Put

$$n=(p_1-1)\dots(p_t-1)km.$$

If we take m sufficiently large, then by Tijdeman's result [2, Theorem 1] and our assumption we can write

$$a_n^n+1=D_iX^k$$

for some $D_i>1$ and we may assume $D_i\neq 2$ by the result of Domar [1]. Taking any prime divisor $p_j\neq 2$ of D_i, by Fermat's theorem we have

$$-1\equiv a_n^n\equiv(a_n^{n/(p_j-1)})^{p_j-1}\equiv 0 \text{ or } 1 \pmod{p_j},$$

which is a contradiction since $p_j\neq 2$.

Proof of Theorem 2 is similar to one of Theorem 1. In this case every

---

Received Oct. 31, 1976

*Division of General Education, Ryukyu University

p_j is not 2.

References

[1] Domar, Y., On the diophantine equation $|Ax^n-By^n|=1$, $n\geqq 5$. Math. Scand., **2** (1954) 29—32 (Mordell, Diophantine Equations § 28 Theorem 12, p. 274)

[2] Tijdeman, R., On the equation of Catalan, Acta Arith. **29** (1976) 179—209.