A Novel Calculation Method for Iron Loss Resistance Suitable in Modeling Permanent-Magnet Synchronous Motors

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Abstract—This paper proposes a calculation method for iron loss resistance, suitable for modeling permanent-magnet synchronous motors. The proposed method is based on the linear feature between semi-input power and square of speed electromotive force. The iron loss resistance is calculated from the slope of this linear function in the offline manner. The advantage of the proposed method is that the iron loss resistance can be calculated directly without any measurements related to mechanical loss. In addition, the proposed method can be executed at any load conditions. The validity of the proposed method is experimentally confirmed by the comparison between the actual torque and the calculated torque containing the iron loss resistance.

Index Terms—Iron loss resistance, modeling, parameter calculation, parameter mismatch, permanent-magnet synchronous motor.

I. INTRODUCTION

In recent years, vector-controlled ac motors, such as induction motor, permanent-magnet synchronous motor (PMSM), and synchronous reluctance motor, have become standard in industrial drives and their performance improvement is an important issue. Particularly, improvement of control performance and drive efficiency is essentially required for drives used in electric vehicles. Conventional vector control strategies have been implemented under the assumption that there is no iron loss in motors. However, in reality, there is a certain amount of iron loss, now small it may be, that influences the flux linkage and output torque of the motors. In order to improve not only the drive efficiency but also control performance, it is necessary to compensate for this iron loss influence on vector-controlled drives [1]. For this reason, several authors have made an attempt to consider the iron loss in vector controlled ac motor drives [2]–[16]. These methods use an equivalent circuit, in which iron loss resistance is inserted in parallel with the magnetizing branch and speed electromotive force (emf). According to this mathematical model, the output torque is strictly proportional to magnetizing currents. From the perspective of improving the torque control, the output torque should be controlled by the magnetizing currents. Since the magnetizing currents cannot be detected directly from the terminal quantities, they are indirectly calculated using the line currents and the iron loss resistance [3], [5], [9], [17]. On the other hand, from the improvement of the drive efficiency point of view, the iron loss resistance is indispensable for the efficiency control strategies [17]–[20]. Thus, it is necessary to compute the iron loss resistance to achieve the accurate vector control performance.

An online identification of the iron loss resistance [11] and adaptive iron loss compensation techniques [12], [13] are developed for induction motor drives. However, in the online identification methods, it is difficult to include mechanical loss. As a result, the accuracy on the iron loss resistance is degraded. It is well known that the iron loss is a part of no-load loss in an ac motor drive and it is obtained by subtracting a mechanical loss from the no-load loss in the offline manner. A general method for dividing the no-load loss into the iron loss and mechanical loss uses the characteristic curve of the no-load loss versus input voltage. In this method, the no-load loss at zero input voltage is assumed to be the mechanical loss and the iron loss is calculated by subtracting the mechanical loss from the no-load loss. However, since the no-load loss at zero input voltage cannot be measured directly, it is estimated by the interpolation of the characteristic curve in low input voltage area. The iron loss resistance is calculated from the obtained iron loss. The disadvantage of this method is that the interpolation is undetermined. Further, any error involved in the measurement of mechanical loss (offset the estimated iron loss) leads to over or underestimation of iron loss resistance. Although this problem can be resolved by using an auxiliary motor [21], it is restricted only to the particular application.

This paper proposes a novel calculation method for the iron loss resistance useful for modeling of PMSM. The iron loss resistance is calculated based on the linear feature between semi-input power and square of speed emf in the offline manner. Here, the semi-input power means the power which is calculated by subtracting the copper loss from the input power (i.e., the semi-input power is equivalent to the sum of the no-load loss and the mechanical output power under load conditions), while the semi-input power is equivalent to the no-load loss only under a no-load condition. As compared with the conventional method, the measurement data are the same (i.e., the input power, input voltage, and input current), while the manipulation of the measurement data is peculiar. First, the semi-input power and the square of speed emf are calculated from the measurement data. Then, the characteristic of the semi-input power versus the square of speed emf is plotted. The characteristic appears as a linear and its slope is equal to the inverse of the
From (4), it can be noted that the iron loss depends on the electrical angular velocity $\omega_e$, and flux linkages ($\Psi_d$ and $\Psi_q$). The flux linkage equations for PMSM are given as

$$\begin{align*}
\Psi_d &= L_i \Psi_{dm} + K_e \\
\Psi_q &= L_i \Psi_{qm}
\end{align*}$$

(5)

where $L_i$ is the armature inductance and $K_e$ is the emf constant. The output torque is calculated from the vector product of the flux linkages and magnetizing currents as

$$\tau = P \left( \Psi_d \Psi_{qm} - \Psi_q \Psi_{dm} \right) = PK_e \Psi_{qm}$$

(6)

where $P$ is the number of pole pairs. As can be seen from (6), the output torque is proportional to the $q$-axis magnetizing current. Thus, it is necessary to control the magnetizing current in order to control the output torque exactly. However, since the magnetizing current cannot be obtained directly from the terminal quantities, this paper uses the following procedure [22].

The magnetizing currents are the difference between the line currents and iron loss currents given by

$$\begin{align*}
i_{dm} &= i_d + \frac{\omega_e L_i}{R_i} i_{qm} \\
i_{qm} &= i_q - \frac{\omega_e L_i}{R_i} \left( i_{dm} + \frac{K_e}{L_i} \right)
\end{align*}$$

(7)

Substituting (7) into (8), gives the $q$-axis magnetizing current as

$$i_{qm} = \frac{1}{1 + \left( \frac{\omega_e L_i}{R_i} \right)^2} \left( i_q - \frac{\omega_e L_i}{R_i} \left( i_d + \frac{K_e}{L_i} \right) \right)$$

(9)

Here, an assumption, iron loss resistance $R_i$ is much greater than reactance $\omega_e L_i$, is used

$$\left( \frac{\omega_e L_i}{R_i} \right)^2 \ll 1$$

(10)

The validity of this assumption is confirmed in Section IV. Applying (10) to (9) results

$$i_{qm} = i_q - \frac{\omega_e L_i}{R_i} \left( i_d + \frac{K_e}{L_i} \right)$$

(11)

Here, the line currents ($i_d$ and $i_q$) can be obtained directly from the terminal quantities, then the $q$-axis magnetizing current can be easily calculated from (11).

On similar lines, the $d$-axis magnetizing current can be obtained, and it is expressed as

$$i_{dm} = i_d + \frac{\omega_e L_i}{R_i} \left( i_q - \frac{\omega_e K_e}{R_i} \right)$$

(12)

As can be seen from (11) and (12), the iron loss resistance $R_i$ is necessary to calculate the magnetizing currents in addition to the conventional PMSMs' parameters, such as the armature inductance $L_i$ and emf constant $K_e$. This paper proposes a calculation method for this iron loss resistance.

II. MATHEMATICAL FORMULATION OF PMSM TAKING IRON LOSS INTO ACCOUNT

In the synchronous reference frame ($d$-$q$), the voltage equations for PMSM are expressed as

$$\begin{align*}
v_d &= R_i i_d + p \Psi_d - \omega_e \Psi_q \\
v_q &= R_i i_q + p \Psi_q + \omega_e \Psi_d
\end{align*}$$

(1)

where the first term on the right-hand side represents the voltage drop for the armature resistance $R_i$, and the second and third terms represent the transformer emf and speed emf, respectively.

Fig. 1 shows the $d$-$q$ axes equivalent circuits of PMSM [18] which are traditionally used when the iron loss is considered. In this circuit, an iron loss resistance $R_i$ is inserted in the parallel fashion. Thus, the $d$-$q$ axes line currents ($i_d$, $i_q$) are divided into iron loss currents ($i_{dtr}$, $i_{qtr}$) and magnetizing currents ($i_{dm}$, $i_{qm}$). In this equivalent circuit, the iron loss $P_i$ due to iron loss resistance is modeled as an equivalent copper loss as

$$P_i = R_i (i_d^2 + i_q^2).$$

(2)

In the steady state, the iron loss currents are expressed as

$$\begin{align*}
i_{dtr} &= -\frac{\omega_e \Psi_q}{R_i} \\
i_{qtr} &= \frac{\omega_e \Psi_d}{R_i}
\end{align*}$$

(3)

Substituting (3) into (2) results in the following equation:

$$P_i = \frac{\omega_e^2 (\Psi_d^2 + \Psi_q^2)}{R_i}$$

(4)
III. CALCULATION METHOD FOR IRON LOSS RESISTANCE

In the steady state \( p = 0 \) and from (1)–(5), the input power \( P_{in} \) is expressed as

\[
P_{in} = v_d i_d + v_q i_q
\]

\[
= R (i_d^2 + i_q^2) + \frac{\omega_c (\Psi_d^2 + \Psi_q^2)}{R_i} + \omega_c K_i i_q
\]

where the first term is the copper loss \( P_c \), the second term is the iron loss \( P_i \), and the third term is the output power \( P_{out} \). According to this formulation, the semi-input power \( P_{si} \), which is defined as the power calculated by subtracting the copper loss from the input power, corresponds to the sum of the iron loss and output power as

\[
P_{si} = P_i + P_{out} = \frac{1}{R_i} \omega_c (\Psi_d^2 + \Psi_q^2) + P_{out}.
\]

Note that the proposed method implicitly includes the mechanical loss and stray loss although the mechanical loss and stray loss are never expressed explicitly.

When both the rotor speed and load torque are constant, the output power \( P_{out} \) is also constant, it is because the mechanical, stray losses, and the pure mechanical output power terms are constants. Then, the semi-input power can be regarded as the linear function of the square of the speed emf \( \omega_c^2 (\Psi_d^2 + \Psi_q^2) \), emphasized in (14). In this situation, the slope of this linear function corresponds to the inverse of the iron loss resistance \( 1/R_i \) and the intercept corresponds to the output power \( P_{out} \).

The iron loss resistance \( R_i \) is calculated, employing the following procedure.

1) Operates the PMSM under constant speed and load conditions.

2) By changing the \( d \)-axis current \( i_d \), set of input power \( P_{in} \), input voltage \( V_{rms} \), and input current \( I_{rms} \) measurements were recorded. Note that the change in \( d \)-axis current will influence only the flux linkage but not the output torque.

3) Using the measured data obtained in step 2, the semi-input power and the square of speed emf are calculated from the following expressions:

\[
P_{si} = P_{in} - R (i_d^2 + i_q^2)
\]

\[
= P_{in} - 3 R R_i^2
\]

\[
\omega_c^2 (\Psi_d^2 + \Psi_q^2) = (v_d - R i_q)^2 + (v_q - R i_d)^2
\]

\[
= V_{rms}^2 - 2 R P_{in} + 3 R_i^2 I_{rms}^2
\]

where \( V_{rms} = \sqrt{v_d^2 + v_q^2} \), \( I_{rms} = \sqrt{i_d^2 + i_q^2} / \sqrt{3} \).

4) The linear characteristic semi-input power versus the square of speed emf is plotted as shown in Fig. 2.

5) The slope of this linear function is obtained with the least squares method. The neighborhood of operating point \( i_d = 0 \) is linearized in order to avoid the influence of the armature resistance mismatch and magnetic saturation. (Discussion is given in Section V.)

6) The iron loss resistance \( R_i \) is calculated from the inverse of the slope obtained in step 5.

The advantage of the proposed method is that the iron loss resistance can be directly calculated without measuring the mechanical loss. In addition, this method can be used at any load, provided the load is kept constant during measurement. The disadvantage of this method is that the parameter mismatch in the armature resistance \( R \). The influence of this armature resistance mismatch and its countermeasures are discussed in Section V.

IV. CALCULATION RESULTS FOR IRON LOSS RESISTANCE

Fig. 3 shows the experimental setup for the proposed method. The specifications of the tested PMSM employed in this experiment are listed in Table I. The electrical input power applied to the tested PMSM is supplied through the voltage source inverter (VSI) [i.e., dc-link voltage, carrier frequency, and dead time are 150 V, 5 kHz, and 5 μs, respectively]. The electrical input power \( P_{in}, \) input voltage \( V_{rms} \), and input current \( I_{rms} \) are measured with the help of a digital power meter (DPM). In order to keep the rotor speed constant, a speed feedback control is used.
Fig. 3. Experimental system.

<table>
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<th>TABLE 1</th>
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<tr>
<td>MOTOR SPECIFICATIONS</td>
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<td>rated power</td>
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<td>rated torque</td>
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<td>rated speed</td>
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<td>armature inductance</td>
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<td>emf coefficient</td>
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<td>number of pole pairs</td>
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Fig. 4 shows the semi-input power versus the square of speed emf for the no-load condition at speed of 2000 r/min when the \( d \)-axis current is changed from +3.5 A to −3.5 A. As can be confirmed from Fig. 4(a), the square of speed emf decreases with decreasing the \( d \)-axis current. Since the square of speed emf is a quadratic equation with variable \( d \)-axis current, variation in the square of speed emf is enlarged by magnetization operation (\( i_d > 0 \)) and reduced by demagnetizing operation (\( i_d < 0 \)). In addition, it confirms that the output torque is almost kept constant irrespective of changing the \( d \)-axis current, it is because the \( q \)-axis current is kept constant. As can be seen from Fig. 4(b), the semi-input power is almost proportional to the square of speed emf. Thus, the characteristic of the semi-input power versus the square of speed emf can be approximate as a linear function. This characteristic is linearized in the neighborhood of nominal operating point for PMSM [i.e., \( i_d = 0 \) (minimum copper loss, \( P_c \), condition)], because the influence of the armature resistance mismatch and magnetic saturation is serious at the extreme demagnetizing and magnetizing points. After linearization, the iron loss resistance \( R_i \), for no-load condition at 2000 r/min is straightforwardly calculated from the slope of this linear function.

Fig. 5 shows the semi-input power versus the square of speed emf for various load conditions at 2000 r/min. All of the characteristics are linear and their slopes are almost the same (i.e., the iron loss resistance is almost the same, irrespective of load conditions). Of course, the intercept increases with increasing load torque, because it corresponds to the output power.

Similar calculations have been made for different rotor speeds from 750 to 3000 r/min. Fig. 6 shows calculated results for the iron loss resistance obtained for different rotor speeds. As can be seen from Fig. 6(a), the iron loss resistance is almost proportional to the rotor speed. The linear characteristic of the iron loss resistance qualitatively agrees with the results obtained in the literature [12]–[14]. Fig. 6(b) shows the square of impedance ratio \( (\omega_i L_i/R_i)^2 \) versus rotor speed. The ratio is calculated by using the iron loss resistance shown in Fig. 6(a). Since the order of the ratio is 10^{-4}, the relation introduced in (10) is valid.

Fig. 7 shows the iron loss \( P_i \) and output power \( P_{out} \) for the flux linkage under the no-load condition. The iron loss is calculated by using (4), while the output power is calculated by subtracting the iron loss from the semi-input power. The iron loss increases with increasing the flux linkage, while the output power is almost constant with respect to flux linkage. Note that, because of the existence of mechanical loss and stray loss, the output power \( P_{out} \) is not equal to zero, despite the no-load condition. It can be confirmed from Fig. 7 that the semi-input power indicated in (14) is appropriately divided into the iron loss and the output power.
V. INFLUENCE OF ARMATURE RESISTANCE MISMATCH

In the proposed method, the armature resistance \( R \) is used in the calculation of both the semi-input power and the square of speed emf. Then, the parameter mismatch of the armature resistance leads to wrong characterization of the semi-input power versus square of speed emf. As a result, error is involved in the calculation of the iron loss resistance. In this section, the influence of the armature resistance mismatch on the calculation of the iron loss resistance is investigated.

When the error for the armature resistance is \( \Delta R \), the semi-input power is expressed as

\[
P_{si}' = P_{in} - 3(R + \Delta R)I_{rms}^2.
\]

From (15) and (17), the calculation error for the semi-input power \( F_{psi} = (P_{si}' - P_{si}) \) is obtained as

\[
F_{psi} = -3\Delta R I_{rms}^2.
\]

It can be seen from (18) that the error \( F_{psi} \) increases with increasing the input current \( I_{rms} \).

On the similar lines, the square of the speed emf with the armature resistance mismatch \( \Delta R \) is expressed as

\[
\omega_c^2(\Psi_d^2 + \Psi_q^2)' = V_{rms}^2 - 2(R + \Delta R)P_{in} + 3(R + \Delta R)^2 I_{rms}^2.
\]

From (16) and (19), the calculation error for the square of speed emf \( F_{\psi_{sse}} = (\omega_c^2(\Psi_d^2 + \Psi_q^2)^2 - \omega_c^2(\Psi_d^2 + \Psi_q^2)) \) is obtained as

\[
E_{\psi_{sse}} = \Delta R \{ 3(2R + \Delta R)I_{rms}^2 - 2P_{in} \}.
\]

It can be seen from (20) that the error \( E_{\psi_{sse}} \) also increases by increasing the input current \( I_{rms} \).

Fig. 8(a) and (b) shows the characteristics of semi-input power versus the square of speed emf for no-load condition and 0.2 N·m at 2000 r/min, respectively. The error \( \pm 10\% \) of the rated armature resistance is considered. As can be expected from (18) and (20), the calculation error increases with increasing input current (i.e., the absolute value of the \( d \)-axis current. As a result, the characteristics are no longer linear. However, the neighborhood of \( i_d = 0 \) operating point yields a linear characteristic. In addition, the slopes are almost the same despite the parameter mismatch. Accordingly, the influence of the armature resistance mismatch can be relieved by linearizing the characteristic in the neighborhood of \( i_d = 0 \) operating point.

VI. VALIDITY OF CALCULATION RESULTS

In order to confirm the validity of the calculation for the iron loss resistance, the calculated torque containing the iron loss resistance is compared with the actual torque. To verify the accuracy of the iron loss resistance, the calculated torque ignoring the iron loss is also plotted.

Fig. 9 shows the electromagnetic torque versus \( q \)-axis current at 2000 r/min. In this figure, measurements denote the actual torque which is obtained with the help of torque transducer. The
two lines denote the calculated torques and these are calculated by

\[ \tau_{\text{cal}} = PK \left\{ i_d - \omega_L \left( i_d + \frac{K}{L} \right) \right\} \]

(21)

where \( R_i = \infty \), when the iron loss is ignored. In all cases, the d-axis current \( i_d \) is kept at zero. As can be seen from Fig. 9, the calculated torque with proposed calculation method (solid line) agrees well with the actual torque, while the calculated torque with \( R_i = \infty \) (dotted line) is larger than the actual torque by 2.6% of rated torque. In respect of other rotor speeds, similar results have been obtained. These comparisons confirm the validity of the proposed calculation method for the iron loss resistance used in modeling the PMSM.

VII. CONCLUSIONS

This paper has proposed a novel calculation method for the iron loss resistance useful for modeling PMSM. The proposed method is based on the linear characteristic between the semi-input power and the square of speed emf. The iron loss is directly calculated from the slope of this linear function in the offline manner. The advantage of the proposed method is that the iron loss resistance can be directly calculated without measuring mechanical loss. In addition, the proposed method can be used for any load conditions. Although the proposed method suffers from the parameter mismatch of the armature resistance, the countermeasure of this problem has also been investigated. The validity of the proposed method has been experimentally confirmed by the comparison between the actual torque and calculated torque containing the iron loss resistance.

REFERENCES

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